N63-20890

NASA TECHNICAL NOTE

NASA TN D-1964

NASA TN D-1964

A SIMULATOR STUDY OF AIRSPACE REQUIREMENTS FOR THE SUPERSONIC TRANSPORT

by Richard H. Sawyer;
Langley Research Center,
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1963

TECHNICAL NOTE D-1964

A SIMULATOR STUDY OF AIRSPACE REQUIREMENTS

FOR THE SUPERSONIC TRANSPORT

By Richard H. Sawyer

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-1964

A SIMULATOR STUDY OF AIRSPACE REQUIREMENTS FOR THE SUPERSONIC TRANSPORT

By Richard H. Sawyer

SUMMARY

An investigation was conducted to study the horizontal airspace requirements for the supersonic transport during changes in heading at a VORTAC station. The vertical deviations from cruising altitude during turning flight were also studied. The investigation was made with use of a fixed-base simulator programed to represent a delta-wing supersonic transport having a canard control. Manual flight control was used to effect changes in heading up to 45° for bank angles of 15°, 30°, and 45°. The tests were conducted at a Mach number of 3.0 at an altitude of 67,000 feet. Measurements were made of the horizontal and vertical displacements from the flight path and of the primary control deflections.

The results of the investigation indicated that the horizontal airspace utilized depends greatly on the type of turn employed. With the conventional method of initiating the turn after passing through the zone of ambiguity over the VORTAC station, heading changes of 45° required bank angles of 45° or greater (considered excessive for commercial transport operations) to keep the flight path within one-half of the present high-altitude airway width of 26 nautical miles. Use of a lead-type turn in which the turn is initiated at a designated slant range ahead of the station decreased the horizontal airspace used to the degree that heading changes up to 45° could be attained with bank angles as low as 15° without exceeding one-half of the airway width. The vertical airspace utilized during the turns also depended on the type of turn employed. In general, the deviations in altitude during the lead-type turns were significantly less than the deviations during conventional-type turns. Pilot workload during the turns, as indicated by the number of control motions required, was significantly less in lead-type turns than in conventional turns. Use of ground-track information presented on an oscilloscope aided the pilot in flaring onto the desired new heading with less overshooting or undershooting, decreased the deviations in altitude during the turns, and reduced the pilot workload in the turns.

INTRODUCTION

For the cruising portion of flight, present continental navigation for civil jet transports makes use of airways defined by a series of segments extending between ground-based radio facilities designated as VORTAC stations. VORTAC stations supply aircraft with the magnetic bearing to the station and slant range from the station. Each segment of the airway is thus defined by the outbound bearing from a given station and the inbound bearing to the next station. In general, at each VORTAC station, a change in heading is required in proceeding along the airway. However, at each station the cone of silence creates a zone of ambiguity in which erratic indications on the heading, distance, and to-from displays exist. Present piloting practice is to proceed on the inbound heading past the station until the zone of ambiguity is cleared before turning to the outbound heading.

Projection of the use of the present navigation aids and piloting technique to the supersonic transport indicates, among several foreseeable difficulties, the problem of airspace required for the airway structure. The turning radius for a bank angle of 30°, for example, is greater than 75 nautical miles at a Mach number of 3. The zone of ambiguity at supersonic-transport cruising altitudes can vary from about 5 to 25 nautical miles in diameter depending on the antenna design. The combined effects of large turning radii and large zones of ambiguity could thus require airways of large width compared with the present high-altitude airway widths of 26 nautical miles.

The purpose of the present investigation was to study the horizontal airspace requirements for changes in heading of the supersonic transport at a VORTAC station and to determine improved operating techniques to reduce both the horizontal airspace requirements and the pilot workload. The studies were made in a fixed-base simulator. Instrument flight under manual control at a Mach number of 3.0 at 67,000 feet was performed by NASA research pilots. Heading changes up to 45° for bank angles up to 45° were used. Three piloting procedures for executing the heading change were investigated. In addition to the study of the horizontal airspace requirements, vertical deviations from cruise altitude during the turn were investigated. Pilot workload problems were noted by examination of the number of control motions used.

SYMBOLS

a speed of sound in air, ft/sec

b wing span, ft

$\bar{c}$ mean aerodynamic chord, ft

C_l rolling-moment coefficient, $M_X/\bar{q}bS$

$C_{l_p} = \partial C_l/\partial(pb/2V)$

$C_{l_r} = \partial C_l/\partial(rb/2V)$

$C_{l_\beta} = \partial C_l/\partial\beta$

$C_{l_{\delta_a}} = \partial C_l / \partial \delta_a$

$C_{l_{\delta_r}} = \partial C_l / \partial \delta_r$

C_m pitching-moment coefficient, $M_Y / \bar{q} S \bar{c}$

$C_{m_\alpha} = \partial C_m / \partial \alpha$

$C_{m_{\delta_c}} = \partial C_m / \partial \delta_c$

$C_{m_{q+\dot{\alpha}}} = \partial C_m / \partial (q\bar{c}/2V) + \partial C_m / \partial (\dot{\alpha}\bar{c}/2V)$

C_n yawing-moment coefficient, $M_Z / \bar{q} b S$

$C_{n_p} = \partial C_n / \partial (pb/2V)$

$C_{n_r} = \partial C_n / \partial (rb/2V)$

$C_{n_\beta} = \partial C_n / \partial \beta$

$C_{n_{\delta_r}} = \partial C_n / \partial \delta_r$

C_N normal-force coefficient, $F_N / \bar{q} S$

$C_{N_\alpha} = \partial C_N / \partial \alpha$

$C_{N_{\delta_c}} = \partial C_N / \partial \delta_c$

C_X longitudinal-force coefficient, $F_X / \bar{q} S$

C_Y side-force coefficient, $F_Y / \bar{q} S$

$C_{Y_\beta} = \partial C_Y / \partial \beta$

$C_{Y_{\delta_r}} = \partial C_Y / \partial \delta_r$

Δd maximum distance off course, nautical miles

F_X longitudinal force, positive forward, lb

F_Y side force, positive to right, lb

F_N normal force, positive downward, lb

g acceleration due to gravity, 32.2 ft/sec^2

h altitude, ft

I_X mass moment of inertia about body X-axis, slug-ft^2

I_Y mass moment of inertia about body Y-axis, slug-ft^2

I_Z mass moment of inertia about body Z-axis, slug-ft^2

l slant range to VORTAC station (lead distance), nautical miles

m mass, slugs

M Mach number, V/a

M_X rolling moment, ft-lb

M_Y pitching moment, ft-lb

M_Z yawing moment, ft-lb

p angular velocity about body X-axis, radians/sec

q angular velocity about body Y-axis, radians/sec

$\bar{q}$ dynamic pressure, $\frac{1}{2}\rho V^2$, lb/sq ft

r angular velocity about body Z-axis, radians/sec

R radius of turn, $V^2/g \tan \phi$, nautical miles

S wing area, sq ft

t time, sec

T thrust, lb

V velocity along flight path, ft/sec

V_X velocity along X-axis, ft/sec

V_Y velocity along Y-axis, ft/sec

V_Z velocity along Z-axis, ft/sec

W weight, lb

X longitudinal body axis

Y lateral body axis

Z vertical body axis

X_i longitudinal inertial (earth) axis

Y_i lateral inertial (earth) axis

Z_i vertical inertial (earth) axis

x_i distance parallel to X_i-axis, nautical miles

y_i distance parallel to Y_i-axis, nautical miles

α angle of attack, V_Z/V, radians

β angle of sideslip, V_Y/V, radians

δ_a aileron deflection, radians

δ_c canard-surface deflection, radians

δ_r rudder deflection, radians

ϵ course angle error, y_i/x_i, radians

θ Euler elevation angle, radians

ρ mass density of air, slugs/cu ft

ϕ Euler roll angle, radians

ψ Euler heading angle, radians

$\Delta\psi$ change in heading, radians

Subscript:

opt optimum for lead or lead-scope turn

A dot over a symbol indicates differentiation with respect to time.

APPARATUS AND METHOD

The supersonic transport was simulated by a modified fixed-base fighter cockpit and analog computer arrangement. The general arrangement of the cockpit is shown in figure 1. The equipment in the cockpit included a single throttle control, a sidearm controller for longitudinal and lateral control, rudder pedals for directional control, flight and navigation instrumentation, and an oscilloscope used to present a simple pictorial display. The sidearm controller was used in order to provide room for installation of the oscilloscope. Details of the flight and navigation instrumentation used are shown in figure 2. The course heading change desired was indicated by grease pencil lines on the face of the oscilloscope as shown in figure 3. The oscilloscope had a face with a high-persistence coating. The actual track followed by the airplane was generated by signals from the analog computer and was retained on the face of the oscilloscope by means of the high-persistence coating.

TABLE 1

CHARACTERISTICS OF HYPOTHETICAL SUPERSONIC TRANSPORT

Physical characteristics:	
W (cruise condition), lb	300,000
$(T/W)_{max}$ (with afterburning, cruise condition)	0.40
I_X, slug-ft^2	2×10^6
I_Y, slug-ft^2	8×10^6
I_Z, slug-ft^2	10×10^6
S, sq ft	4,300
b, ft	90
$\bar{c}$, ft	64
Aerodynamic characteristics:	
C_X	$0.0085 - 0.239\alpha^2$
C_m	$C_{m_\alpha}\alpha$
C_N	$C_{N_\alpha}\alpha$
C_{l_p}	-0.111
C_{l_r}	0.01
C_{l_β}	-0.05
$C_{l_{\delta_r}}$	0.01
$C_{l_{\delta_a}}$	0.02
$C_{m_{\delta_c}}$	0.5
$C_{m_{q+\dot{\alpha}}}$	-2.0
C_{n_β}	0.05
$C_{n_{\delta_r}}$	-0.01
C_{n_r}	-1.67
C_{n_p}	0.02
$C_{Y_{\delta_r}}$	0.1
C_{Y_β}	-0.5
$C_{N_{\delta_c}}$	0.05
C_{N_α}	1.383
C_{m_α}	-0.086
Control characteristics:	
Deflection range -	
Aileron, radians	±0.4
Canard surface, radians	±0.04
Rudder, radians	±0.5
Power -	
$(pb/2V)_{max}$, radians	0.07
δ_c/g, radians/g	0.0126

TABLE 2

CRUISING CONDITIONS

h, ft	67,000
M	3.0
T/W (with partial afterburning)	0.14
$\bar{q}$, lb/sq ft	686
ρ, slugs/cu ft	0.0001615
α, radians	0.069
$\delta_{c,trim}$, radians	0.012

The selected characteristics of the supersonic transport programed in the analog computer are given in table 1. The physical characteristics represent a delta-wing canard-type design. The values of the damping-in-yaw and damping-in-pitch parameters C_{n_r} and $C_{m_{q+\dot{\alpha}}}$ are augmented values required to provide satisfactory handling qualities. The conditions of cruising flight are given in table 2.

The six-degree-of-freedom equations of motion used in the analog computer programing are given in the appendix. The classical equations have been simplified by elimination of certain aerodynamic coefficients and cross-coupling inertia terms whose effects have been shown to be negligible or of secondary importance (ref. 1). The conventional relationship of the aircraft body axes, Euler angles, and inertial axes was used for derivation of these equations (fig. 4). In addition, because the excursions in Mach number and angle of attack were expected to be small, the aerodynamic coefficients, with the exception of the longitudinal-force coefficient, were assumed

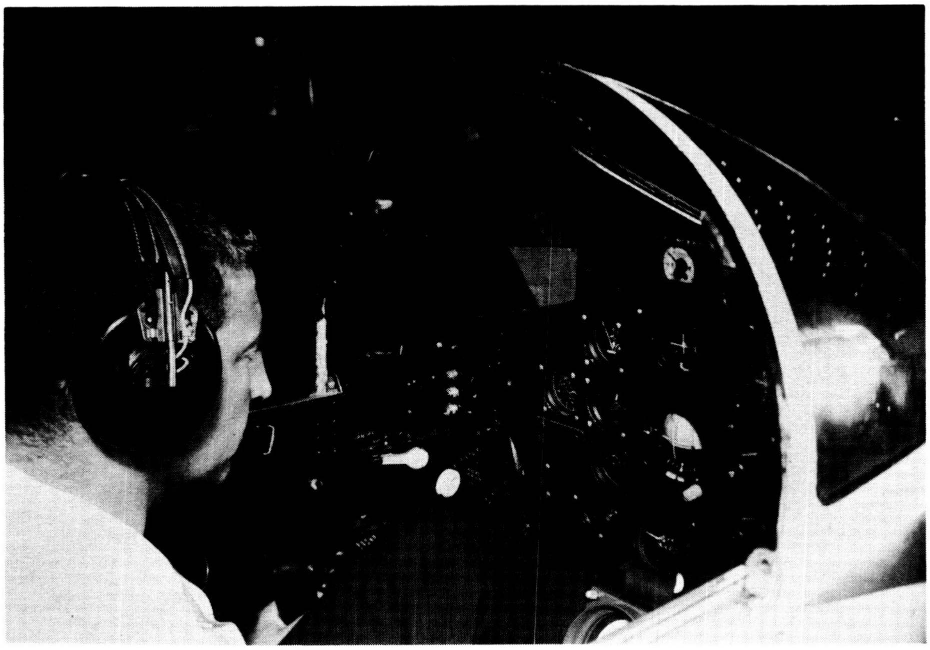


Figure 1.- General arrangement of cockpit. L-61-3800

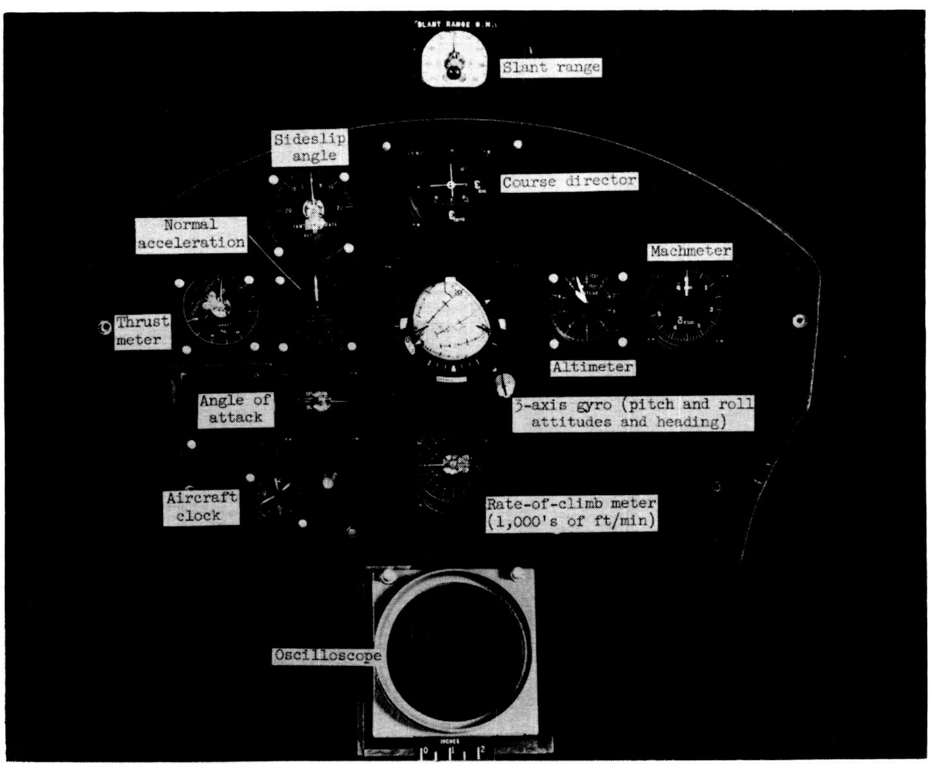


Figure 2.- Flight and navigation instrumentation. L-61-4423.1

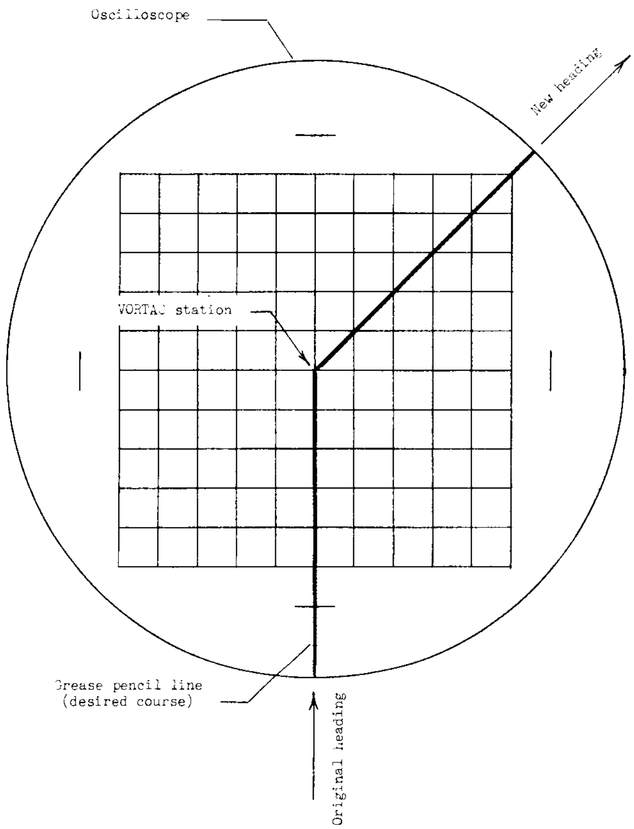


Figure 3.- Schematic representation of method of indicating course-heading change desired on face of oscilloscope.

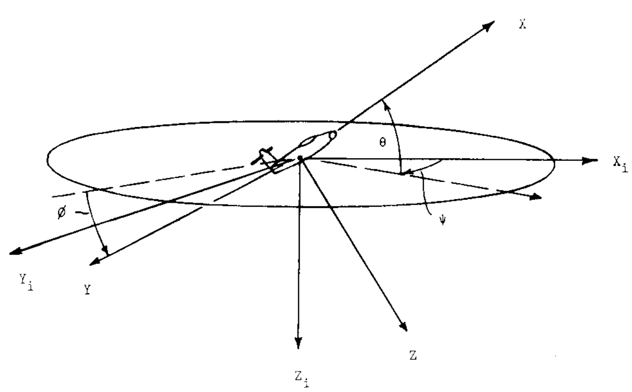


Figure 4.- Diagram of Euler angles, body axes, and inertial axes.

invariant. The longitudinal-force coefficient was assumed to vary with angle of attack as is indicated in table 1. The angles of attack, sideslip, and course error were defined (see symbols section) by using small angle approximations. The thrust axis was assumed alined with the longitudinal body axis. The signals required to operate the flight panel instruments were developed in the analog computer.

TEST PROCEDURE AND TESTS

Each test run was initiated at a Mach number of 3.0 at an altitude of 67,000 feet on a northerly heading approximately 80 nautical miles due south of the simulated VORTAC station. The piloting procedure was to fly along the 180° radial to the station, make the required heading change at a specified bank angle, and terminate the run when the airplane was stabilized on the new course. Mach number and altitude were maintained as close to the initial conditions as possible. Heading guidance was obtained from the radio magnetic-compass mode of the three-axis directional gyro instrument, the course-director instrument (a "fly-to" indicator using a null signal to indicate on course), and the pictorial display generated on the oscilloscope. In general, the pilot used the least sensitive response setting on the course-director instrument when far from the station, the intermediate sensitivity at the intermediate distances, and the greatest sensitivity when near the station.

Test runs were made for heading changes of 5°, 15°, 30°, and 45°. For each heading change, bank angles of 15°, 30°, and 45° were used. All heading changes were from the north toward the east. Seven pilots participated in the program, with the individual participating level (including practice) ranging from 5 to 41 runs.

The specified heading changes were made by using two different turn procedures. Typical of present piloting practices, one set of turns was made by initiation of the turn after passing through the zone of ambiguity. A second set of turns was made by initiation of the turn at a predetermined slant range ahead of the station calculated on the basis of the speed, altitude, heading change, and bank angle by means of the following expression:

$$l = \sqrt{\left(\frac{V^2}{g \tan \phi} \tan \frac{\Delta\psi}{2}\right)^2 + h^2}$$

The slant range (lead distance) thus calculated is for a flight path along an arc of a circle which will bring the aircraft tangent to the new heading. (See fig. 5.) In order to classify the type of turn maneuver, the turn initiated at or past the station will be referred to as a conventional turn, and the turn initiated ahead of the station will be referred to as a lead turn. Since some of the lead turns were made by using the oscilloscope to provide the pilot with ground-track information, this type of turn will be referred to as a lead-scope turn.

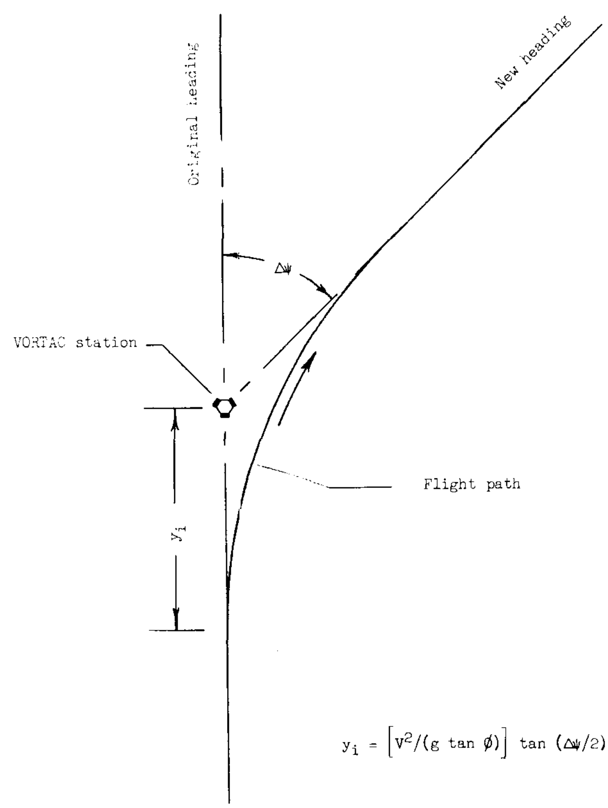


Figure 5.- Schematic representation of calculated flight paths using lead-turn method.

The zone of ambiguity over the VORTAC station was simulated by causing erratic indications on the course-director and slant-range meters; these erratic indications were generated by the addition of a white noise to the signals to these instruments. The zone of ambiguity was assumed to have a diameter of 22 nautical miles at the cruise altitude, which is representative of an antenna with a cone-of-silence angle of 90°. During the passage through the zone of ambiguity, the incremental signal required to null the course-director needle on the new course was applied by the operator.

The ground track of the airplane during each run was plotted on a servo plotting board at a scale of 16.44 nautical miles to the inch. Time-history recordings of the quantities listed in table 3 were made on two multichannel direct-writing rectilinear magnetic recorders at the sensitivities given.

TABLE 3

QUANTITIES AND RECORDING SENSITIVITY

Quantity	Recording sensitivity
α	0.955 deg/mm
β	0.286 deg/mm
δ_c	0.076 deg/mm
δ_r	0.955 deg/mm
δ_a	1.528 deg/mm
p	0.100 radian/sec/mm
q	0.040 radian/sec/mm
r	0.004 radian/sec/mm
l	8.22 nautical miles/mm
$\Delta\psi$	4.00 deg/mm
ΔM	0.04/mm
ϵ	5.73 deg/mm
$\Delta(T/W)$	0.002/mm
F_N/m	0.20 g/mm
Δh	200 ft/mm
ϕ	4.00 deg/mm

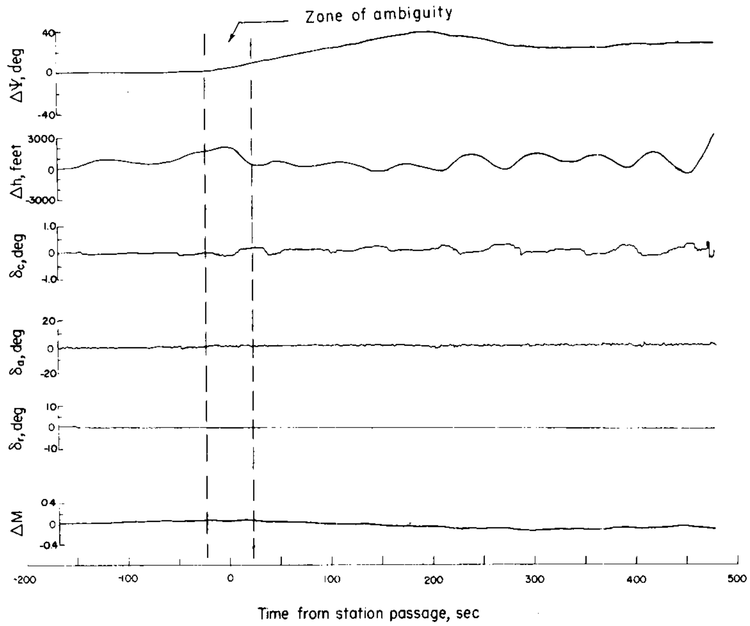
RESULTS AND DISCUSSION

Typical Results

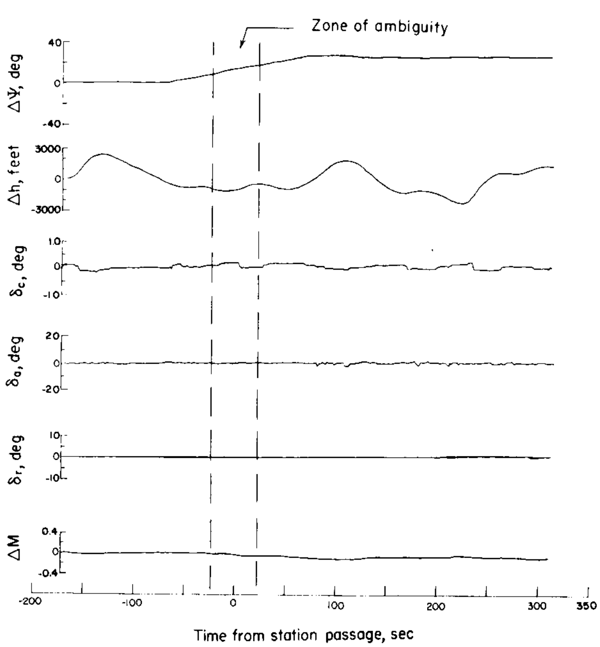
Typical time histories of the pertinent quantities recorded are given in figure 6 for conventional, lead, and lead-scope turns. Illustrative ground-track plots for the same type of turns are given in figure 7. Comparison of the time histories (fig. 6) indicates that the lead and lead-scope turns require less time than the conventional turn to complete the turn, and require fewer and smaller corrections to the initial change in heading in order to complete the required change in heading. Furthermore, comparison of the ground-track plots (fig. 7) indicates that the lead and lead-scope turns consume less horizontal airspace than the conventional turn.

Pilot Opinions

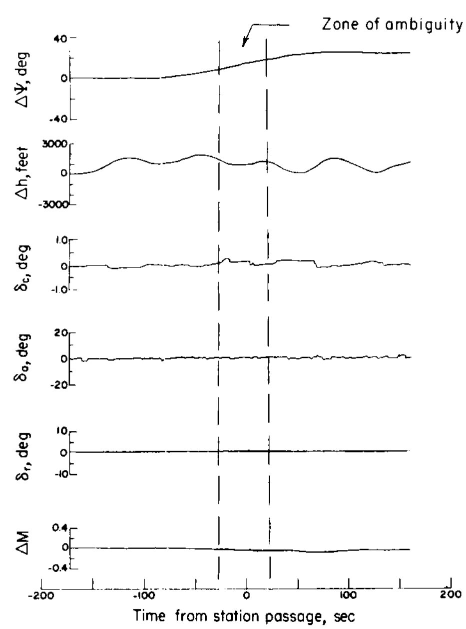
Preference for the lead turn over the conventional turn was voiced by the pilots, particularly on the basis of the reduction in number of maneuvers required to make the turn and the simplification of the maneuvers. The lead turn, for instance, basically requires only a roll into the turn and a roll out of the turn. The conventional turn in contrast basically requires a roll into the turn, a roll out on a cut-back course to approach the desired course, a roll into an opposite turn, and a roll out on course. The coordination of these various maneuvers is difficult with the result that overshoots and undershoots and resulting corrections occurred more frequently and were of greater magnitude for the conventional turn than for the lead turn. With regard to the lead-scope turns, the pilot felt that having the ground-track information on the scope aided them considerably in flaring onto the desired course with less overshooting or undershooting.



(a) Conventional turn.



(b) Lead turn.



(c) Lead-scope turn.

Figure 6.- Typical time histories of changes in heading, altitude, and Mach number in turns. Aircraft control motions also shown.

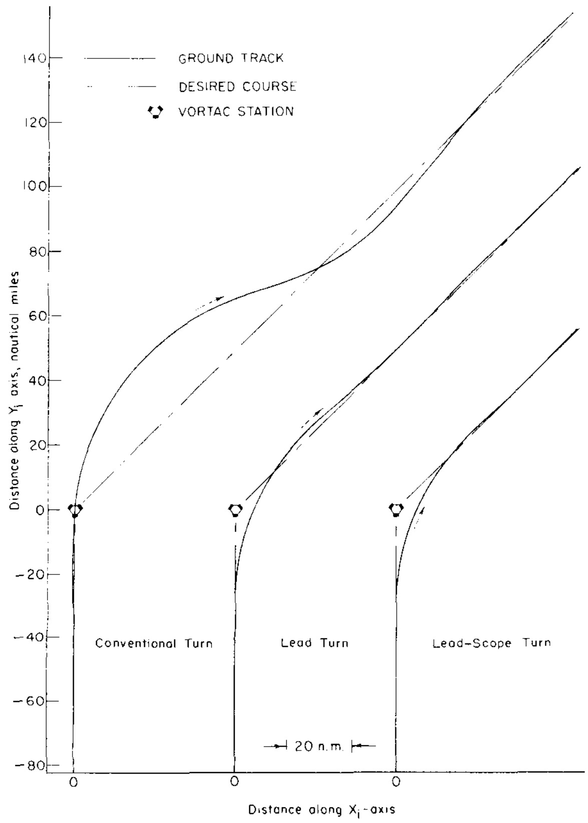


Figure 7.- Typical ground-track plots in turns.

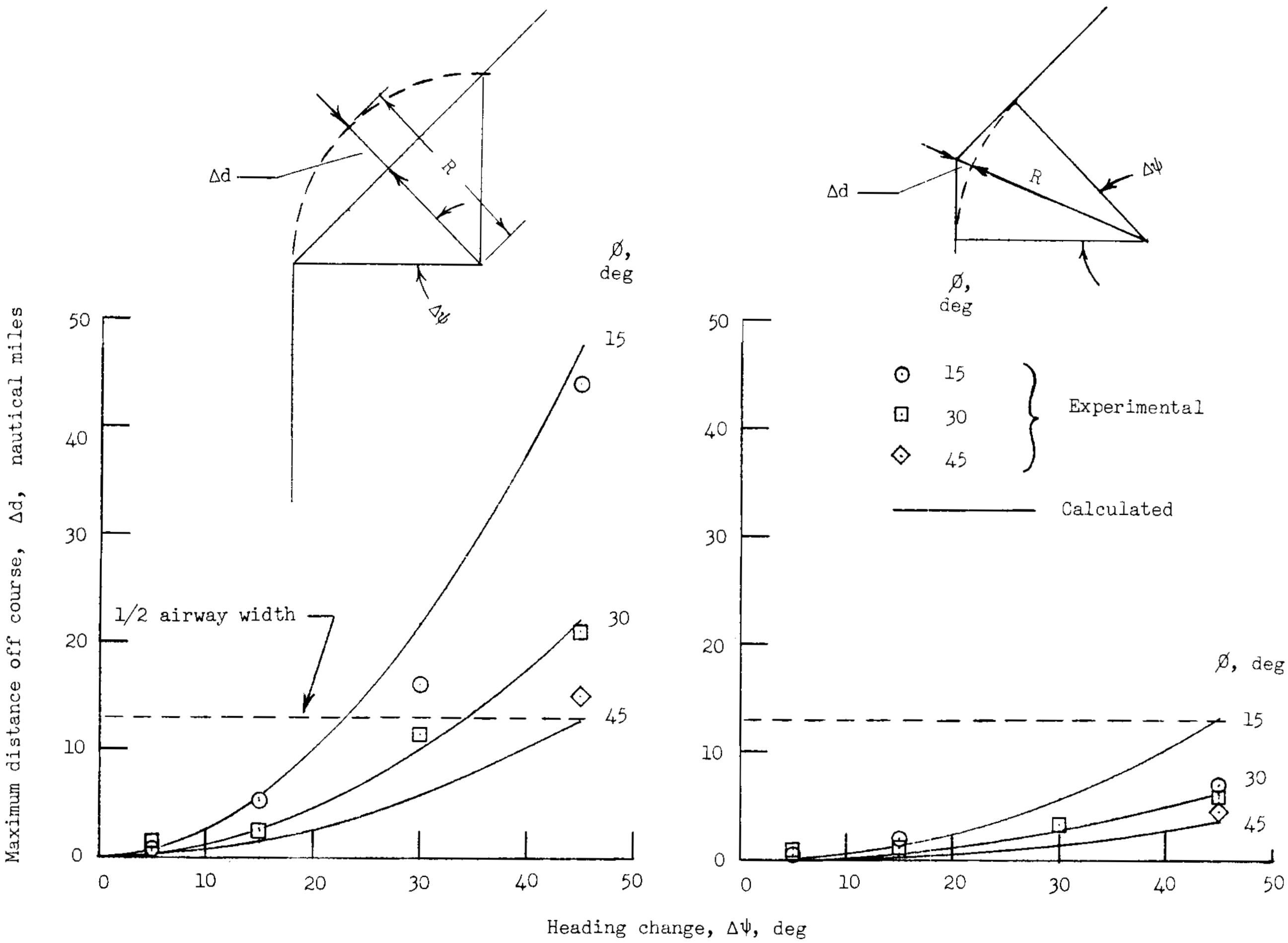
Horizontal airspace in turns.- The maximum distance off course Δd is shown plotted against heading change for bank angles of 15°, 30°, and 45° in figure 8. Both experimental and calculated values are given for conventional, lead, and lead-scope turns. The experimental results represent the average value of several turns at the specified conditions, and the average includes, in general, results obtained with more than one pilot. The calculated results were obtained by use of the following expressions (see fig. 8):

$$\Delta d = R(1 - \cos \Delta\psi) \qquad \text{(conventional turn)}$$

$$\Delta d = R\left(\sec \frac{\Delta\psi}{2} - 1\right) \qquad \text{(lead turn)}$$

The results shown in figure 8 show good agreement in general between the experimental and calculated values of the maximum distance off course, which would indicate that calculated values are satisfactory indices of the deviations from course during turns for supersonic flight at high altitudes. The results shown in figure 8(b) are for both lead and lead-scope turns as no significant effect of using the scope was apparent. The results show large reductions in the maximum distance off course for the lead and lead-scope turns compared with the conventional turns, with the greatest reductions occurring for the larger heading changes. Since the airway width for the altitude range from 30,000 feet to 75,000 feet is 26 nautical miles, it appears that by use of either the lead or lead-scope turn method, the deviation from the center line of the course can be held within the airway width for heading changes up to 45° with bank angles as low as 15°. For the conventional turn, the airway boundary would be exceeded for changes in heading as low as 23° with a bank angle of 15°, and only by use of an excessive bank angle of 45° could a heading change of 45° be accomplished within the airway width.

Altitude deviations in turns.- The amount of time the aircraft exceeded 200-foot increments in altitude from the cruise altitude during the turns is shown in figure 9. The amount of time is expressed as a ratio based on the time to complete the optimum lead turn. The results represent the total time for both positive and negative altitude deviations. Results are given for conventional, lead, and lead-scope turns for several heading changes and several bank angles. Each plot includes the results for all pilots making the particular turn. For the



(a) Conventional turn.

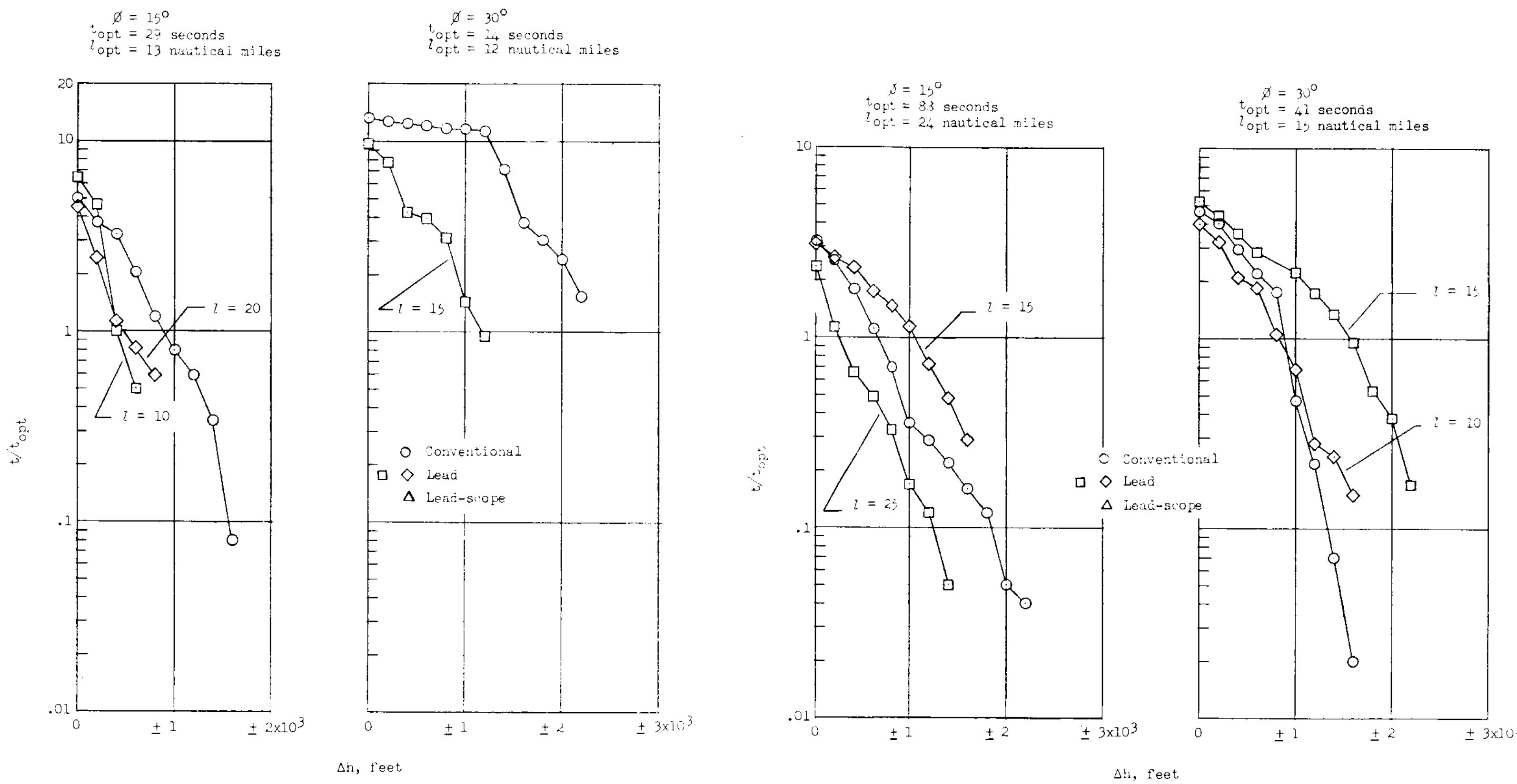
(b) Lead and lead-scope turns.

Figure 8.- Variation of maximum distance off course with heading change for several bank angles. Conventional, lead, and lead-scope turns.

lead turns, results are given both for turns using the optimum or nearly optimum lead distance and for turns using shorter or longer lead distances than the optimum.

Examination of the results in figure 9 shows that even for turns using approximately the optimum lead distance, the amount of time required to complete the turn was often several times the optimum time. This result was due to the fact that the turn was not considered complete until the airplane was stabilized on course at the correct altitude and Mach number, and this stabilizing period after the initial roll-out from the turn took considerable time to effect.

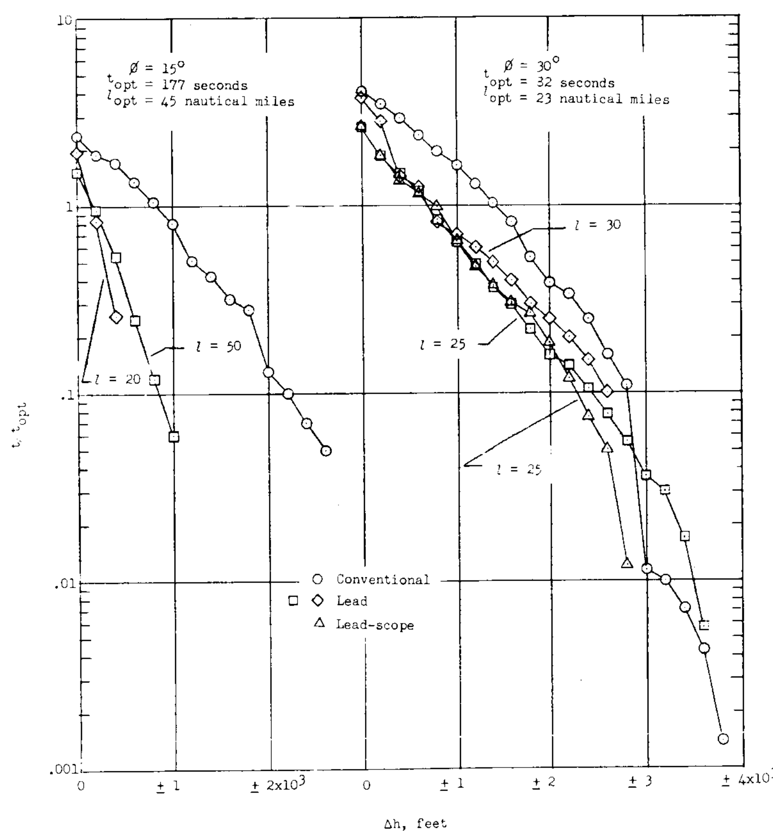
Comparison of the results in figure 9 for conventional, lead, and lead-scope turns indicates that, in general, the use of a lead or lead-scope turn of approximately the optimum lead distance reduces the amount of time at which each 200-foot



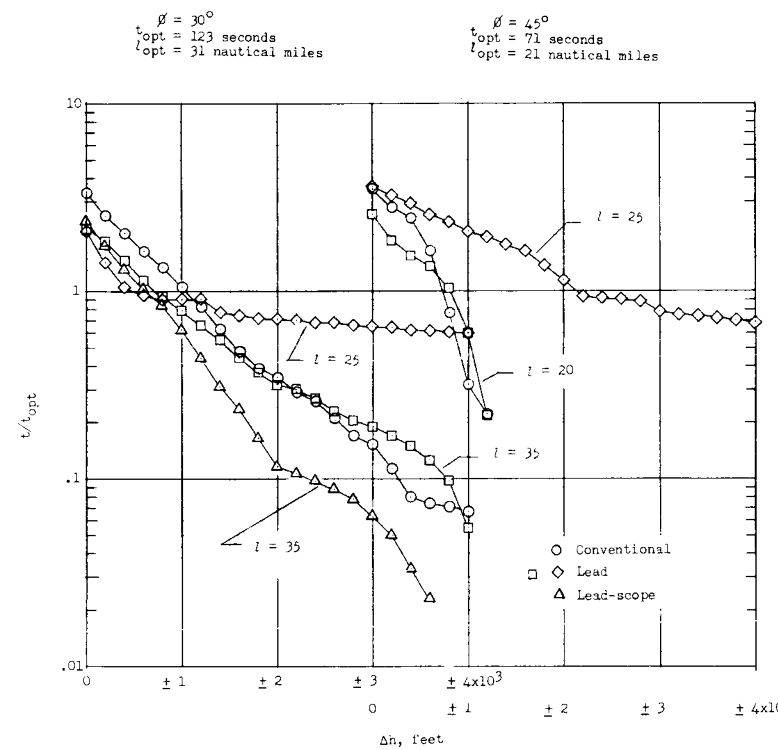
(a) $\Delta\psi = 5^\circ$.

(b) $\Delta\psi = 15^\circ$.

Figure 9.- Time airplane exceeded 200-foot increments in altitude from cruise altitude during conventional, lead, and lead-scope turns.



(c) $\Delta\psi = 30^{\circ}$.



(d) $\Delta\psi = 45^{\circ}$.

Figure 9.- Concluded.

increment in altitude from cruise altitude is exceeded, particularly for the larger deviations in altitude. However, in general, the use of a lead distance other than optimum results in a somewhat larger amount of time at which each altitude increment is exceeded than that of the conventional turn.

For both conventional and lead turns, the general detrimental effect on altitude control of the use of too high a bank angle, particularly for the smaller changes in heading, can be seen by comparison of the results in figure 9 for the two bank angles for each heading change. Recognition by the pilots of the detrimental effects on airspace utilization or on their ability to control the turn due to the use of too high or too low a bank angle for a desired change in heading led to the omission from the program of changes in heading of 5°, 15°, and 30° at a bank angle of 45° and the omission of a change in heading of 45° at a bank angle of 15°.

Results for lead-scope turns are given in figure 9 only for $\Delta\psi = 30^{\circ}$ and $\Delta\psi = 45^{\circ}$ (at $\phi = 30^{\circ}$) as it was determined in a few tests that the oscilloscope was not particularly useful for small changes in heading. The limited results obtained by using the oscilloscope indicate improvement in the ability of the pilots to maintain altitude when using this equipment. In addition, as indicated previously, the pilots expressed a preference for having such ground-track information for ease in establishing the new heading.

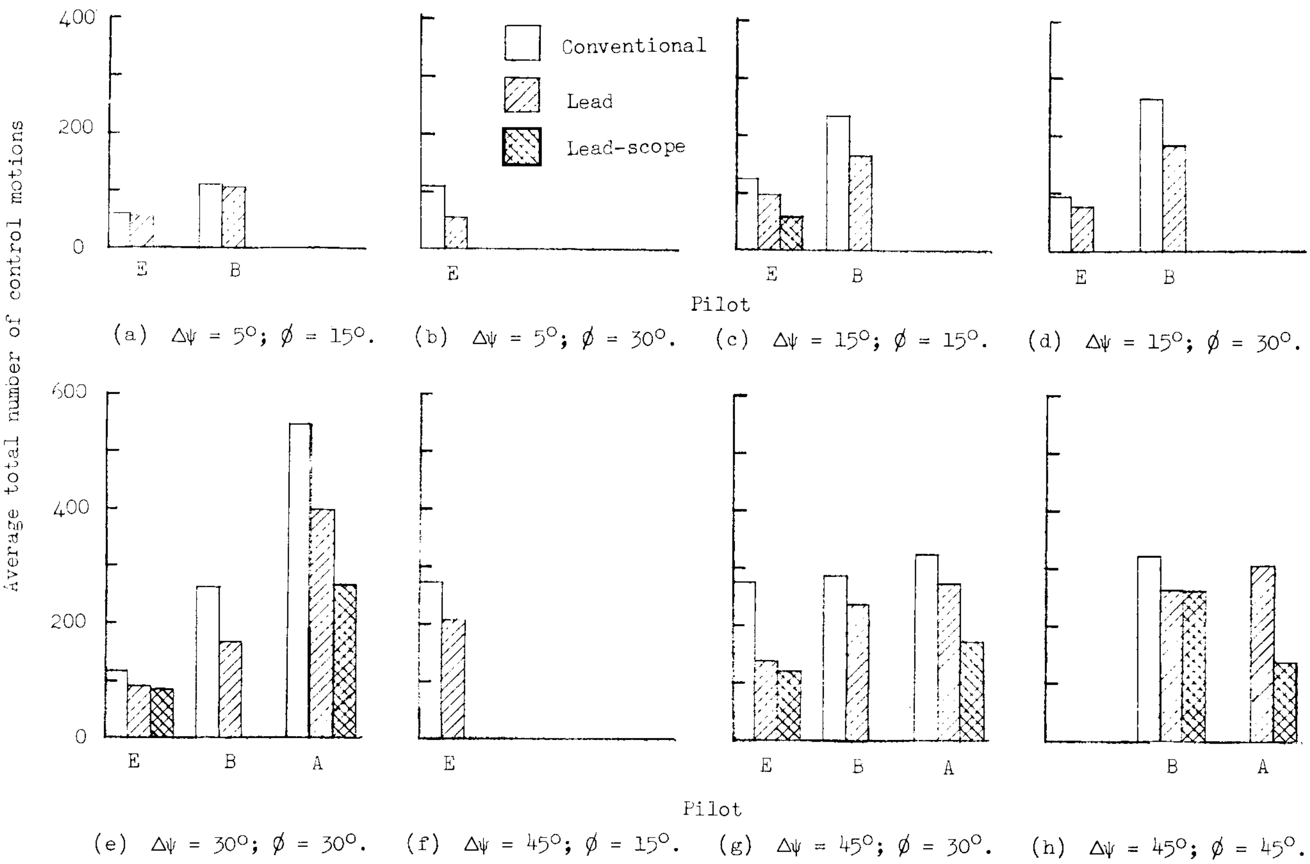


Figure 10.- Average total number of control motions in making conventional, lead, and lead-scope turns. Pilots A, B, and E.

Pilot Workload

In order to indicate the relative workload level for the pilot in making the turns, the total number of control motions (aileron plus canard) was determined for each turn. In this analysis, a control motion was considered to be a movement of the control to a new level, which was held for a significant amount of time, or a movement of the control to a point of control reversal. For each pilot, the total number of control motions used in making turns of a given heading change and bank angle was averaged. The results of the average total number of control motions for the three pilots with the highest total time are given in figure 10 for conventional, lead, and lead-scope turns. It is interesting to note the differences between pilots. Pilot E generally used the least number of motions; pilot B, a larger number; and pilot A, the greatest number. However, the most significant fact of this analysis is the indication for all three pilots of a general decrease in the pilot workload for the lead turns over that for the conventional turns and the further general decrease in pilot workload effected by use of the oscilloscope.

SUMMARY OF RESULTS

The results of fixed-base-simulator tests of the airspace utilized during changes in heading at a VORTAC station of a supersonic transport cruising at a Mach number of 3.0 under manual control are as follows:

1. The horizontal airspace utilized depended greatly on the type of turn employed. With the conventional method of initiating the turn after passing through the zone of ambiguity over the VORTAC station, heading changes of 45° required bank angles of 45° or greater (considered excessive for commercial transport operations) to keep the flight path within one-half of the present high-altitude airway width of 26 nautical miles. Use of a lead-type turn in which the turn is initiated at a designated slant range ahead of the station decreased the horizontal airspace used to the degree that heading changes up to 45° could be attained with bank angles as low as 15° without exceeding one-half of the airway width.

2. The vertical airspace utilized during the turns also depended on the type of turn employed. In general, the deviations in altitude during the lead-type turns were significantly less than the deviations during conventional-type turns.

3. Pilot workload during the turns as indicated by the number of control motions required, was significantly less in lead-type turns than in conventional turns.

4. Use of ground-track information presented on an oscilloscope aided the pilot in flaring onto the desired new heading with less overshooting or undershooting, decreased the deviations in altitude during the turns, and reduced the pilot workload in the turns.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., June 26, 1963.

APPENDIX

EQUATIONS OF MOTION

The six-degree-of-freedom equations of motion used in the analog computer programing are given below. These equations are based on body axes and, in addition to the assumptions indicated in the section entitled "Apparatus and Method," include the following additional assumptions:

(1) The curvature of the earth's surface and the translation of the earth in space are neglected.

(2) The relative motion between the earth and its atmosphere is assumed to be zero.

(3) The airframe is regarded as a rigid body.

(4) The mass, moments of inertia, and products of inertia are time invariant. A fixed center of gravity is assumed.

(5) The X and Z body axes define a plane about which the aircraft is symmetrical.

The definition of the Euler angles describing the orientation of the body axes in space, in terms of the angular velocities of the aircraft about the body axes, and the equations defining the transformation of body-axis velocity components to inertial-axis velocity components are also given below.

Equations of Motion

$$\dot{V}_X = -g \sin\theta + (1/m)\left(-C_X\bar{q}S + T\right) - V_Z q + V_Y r$$

$$\dot{V}_Y = g \cos\theta \sin\phi + \left(\bar{q}S/m\right)\left(C_{Y_{\delta_r}}\delta_r + C_{Y_\beta}\beta\right) - V_X r + V_Z p$$

$$\dot{V}_Z = g \cos\theta \cos\phi - \left(\bar{q}S/m\right)\left(C_{N_\alpha}\alpha + C_{N_{\delta_c}}\delta_c\right) - V_Y p + V_X q$$

$$\dot{p} = \left(\bar{q}Sb/I_X\right)\left[C_{l_\beta}\beta + C_{l_{\delta_r}}\delta_r + C_{l_{\delta_a}}\delta_a + (b/2V)\left(C_{l_p}p + C_{l_r}r\right)\right]$$

$$\dot{q} = \left(\bar{q}S\bar{c}/I_Y\right)\left[C_{m_\alpha}\alpha + C_{m_{\delta_c}}\delta_c + (\bar{c}/2V)C_{m_{q+\dot{\alpha}}}q\right] + \left[\left(I_Z - I_X\right)/I_Y\right]pr$$

$$\dot{r} = \left(\bar{q}Sb/I_Z\right)\left[Cn_\beta \beta + Cn_{\delta_r}\delta_r + (b/2V)\left(Cn_r r + Cn_p p\right)\right] + \left[\left(I_X - I_Y\right)/I_Z\right]pq$$

Euler-Angle Definitions in Terms of Angular Velocities

About Body Axes

$$\dot{\theta} = q \cos \phi - r \sin \phi$$

$$\dot{\psi} = \frac{q \sin \phi + r \cos \phi}{\cos \theta}$$

$$\dot{\phi} = p + \dot{\psi} \sin \phi$$

Transformation of Body-Axis Velocity Components to

Inertial-Axis Velocity Components

$$V_{X_i} = V_X \cos \theta \cos \psi + V_Y(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + V_Z(\sin \phi \sin \psi + \cos \phi \cos \psi \sin \theta)$$

$$V_{Y_i} = V_X \cos \theta \sin \psi + V_Y(\cos \phi \cos \psi + \sin \phi \sin \theta \sin \psi) + V_Z(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)$$

$$V_{Z_i} = -V_X \sin \theta + V_Y \sin \phi \cos \theta + V_Z \cos \phi \cos \theta$$

REFERENCE

1. Connelly, Mark E.: Simulation of Aircraft. Tech. Rep. NAVTRADEVCEN 7591-R-1, U.S. Naval Training Device Center (Port Washington, N.Y.), Feb. 1958. (Reprinted May 1959.)